The Light of Einstein

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Abstract

The Michelson-Morley experiment suggests the hypothesis that the *two-way* speed of light is constant, and this is consistent with a more general invariance than that of Lorentz. On adding the requirement that physical laws have the same form in all inertial frames, as Einstein did, the transformation specializes to that of Lorentz.

1. Introduction

One hundred years ago, in 1905, a 25-year-old employee of the Swiss patent office, Albert Einstein, published five papers in the journal Annalen der Physik that were to have momentous repercussions in the world of physics, and far beyond that cloistered milieu. One paper looked back to the nineteenth century, insofar as it had to do with the atomic theory, the hypothesis that matter is made of molecules that are themselves assemblages of atoms. The remaining four papers formed the very roots of twentieth century physics, relativity and quantum theory. In two of these articles, Einstein made the apparently preposterous proposal that light is corpuscular, as Newton had suggested it might be, despite all the contrary evidence that it has a wave nature. These papers were the second and third hesitant steps towards what is now called 'the old quantum theory', Max Planck having taken the first one in 1900. The remaining two papers of Einstein, in his *annus mirabilis* of 1905, dealt with the theory of relativity. In the first of them, absolute time was knocked from its Newtonian pedestal: after Einstein's paper, it would be recognized that the experience of time is different for observers in relative motion. Already in this first paper, the young Einstein recognized the possibility of what became known as the twin paradox. Arguably the most famous equation in physics, $E = mc^2$, was deduced in the second of these papers as a consequence of the theory of special relativity, although it was not written down in quite that form in this seminal article.

Poincaré in France, Fitzgerald in Ireland and Lorentz in the Netherlands had come close to the idea of the relativity of space and time. In 1904 the last named physicist had in fact published what we still call the Lorentz transformation from a study of the invariance properties of Maxwell's electromagnetic equations. Einstein himself derives this transformation independently, not from Maxwell's equations, but as a consequence of two principles: first that the laws of physics should have the same form in all inertial frames, and second that the measured speed of light in vacuo should be the same in all inertial systems. Only after having obtained the transformation does he show it to describe an invariance of Maxwell's equations.

In the nineteenth century, the general view of light was that it is an electromagnetic wave, propagating in the luminiferous aether, or ether. The Michelson-Morley experiment is commonly taken to show that the measured speed of light in a vacuum, c, is independent of the speed of an observer with respect to the ether; but, strictly speaking, since the measurements involved light that travels from a point to a mirror and then back to the point of emission, the constancy in question is of the average speed forwards and backwards, or for short the two-way speed of light. It is not excluded that light could have a speed greater than c in one direction, and less than c in the other, on condition that the mean is c. Einstein assumed explicitly that the one-way speed is c, but in the next section we propose to impose only the weaker constraint of the constancy of the two-way speed of light. We shall find that there are more possibilities than those inherent in the Lorentz transformation. In particular, the relativity of time and the frame-dependence of simultaneity are not necessary consequences of a constant two-way light speed. They *do follow necessarily* from Einstein's thesis of a constant one-way light speed, and this thesis has been vindicated *a posteriori* by Nature.

The treatment is pedagogical rather than historical: the intention is to lead the reader through the steps that Einstein could have taken, but did not. Starting from the null result of the Michelson-Morley experiment, the calculational details are given in a manner that allows for easy checking by the reader, first with a return path for light in the direction of the observer's motion through the ether, then with a path transverse to this direction, and finally with a general path that is skewed with respect to the direction of the observer's motion.

2. Two-Way Speed of Light

One might naively think that, if the observer is travelling through the ether, and light moves back and forth in the same direction, it will appear to the observer to be moving faster in one direction, and slower in the other, but that the average relative speed will be the same as if the observer were at rest. However, this is not right, for while the propagation time of the light is indeed smaller in one direction, and larger in the other, the cancellation between the two effects occurs only to first order in v/c, not to second and higher orders.

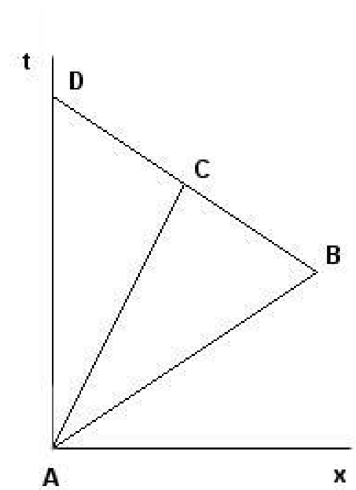

Figure 1: Spacetime Diagram

To simplify the notation, choose units of space and time such that $c = 1$. Suppose that the coordinate system $S = (t, x, y, z)$ is at rest in the ether, and that light travels in all directions at unit speed, as measured from S. A light signal starts from the origin at time zero. The signal travels along the x-axis and arrives at the space-point x_B at time $t_B = x_B$, where it is reflected back along the x-axis to the origin, as shown in the spacetime diagram of Fig. 1. The light arrives back at the origin at time $t_D = t_B + x_B = 2x_B$. Let the spacetime point C lie at the intersection of the line BD and the line given by $x = vt$. The latter is the worldline of a body travelling at a constant velocity v with respect to the origin. Since the speed of light from B to D is also unity, we have

$$x_B - x_C = t_C - t_B \ .$$

Insert $t_B = x_B$ and $x_C = vt_C$ into this equation to obtain

$$t_C = 2x_B / (1+v) \qquad\qquad x_C = 2vx_B / (1+v) \ . \tag{1}$$

We ask now how an observer, travelling at a constant velocity v along the x-axis, would interpret the spacetime points A, B and C. For her the light is emitted at the origin A, reflected at B, but returns to the comoving origin at C, not D. For this observer, the distance travelled by the light from A to B is x'_B , and the distance travelled back to her origin, C, is $x'_B - x'_C = x'_B$, since $x'_C = 0$ is the comoving origin for the observer. To calculate displacements, as seen by the moving observer, we have the Galilean transformation from S to S' :

$$x' = x - vt \ , \tag{2}$$

from which it follows that, if $x = vt$, then $x' = 0$. What would our moving observer find for the return trip speed of light, from A to B and back to C? She will estimate the distance from A to B as

x'_B , and the distance back to C also as x'_B . The total time for the round trip is t_C , which can be read off from Eq. (1). She finds for the average speed

$$< c > = 2\, x'_B / t_C = 2\,(x_B - vt_B)\,(1 + v) / (2\, x_B) = 1 - v^2 , \qquad (3)$$

which is less than unity, i.e. less than light-speed. This is in contradiction to the experimental finding that the two-way average speed of light is constant, i.e. it is independent of the velocity of the observer with respect to the ether.

To repair this defect, we replace the Galilean transformation (2) by

$$x' = \alpha\,(x - vt) , \qquad (4)$$

where α is a parameter that may depend on v, but not on t or x. This still respects the requirement that, if $x = vt$, then $x' = 0$. It is not necessary for α to be unity, as it is for a Galilean transformation. This factor will give rise to a contraction or scale-change in the distances measured in the direction of the motion. The average speed for the return trip is now

$$< c > = 2\, x'_C / t_C = \alpha\,(1 - v^2) , \qquad (5)$$

so if we set

$$\alpha = 1 / (1 - v^2) , \qquad (6)$$

the average speed will be unity, just as it is for an observer who is not moving. Note that this works for any observer, moving with any constant velocity along the x-axis, on condition that the transformation (4) describes his coordinate, with the corresponding coefficient (6) adjusted to his velocity with respect to the ether. In particular, no departure from Newtonian absolute time was necessary to achieve this result.

To see that the transformation given by Eqs. (4)-(6) does not suffice to ensure the constancy of the measured two-way velocity of light in other directions, consider a light signal that travels from the origin to the point F, where it is reflected back symmetrically to the x-axis at the point G, as shown in the space diagram of Fig. 2. If $x_F = vt_F$, $x_G = 2vt_F$ and $y_G = 0$, then for the observer travelling with velocity v along the x-axis, the light is transmitted along her y'-axis, and then is reflected back along it, arriving at the point G when she herself reaches that point. We have $x_F^2 + y_F^2 = t_F^2$, and thus $t_F^2 = y_F^2 / (1 - v^2)$. From the point of view of the moving observer, the total distance traversed by the light signal is

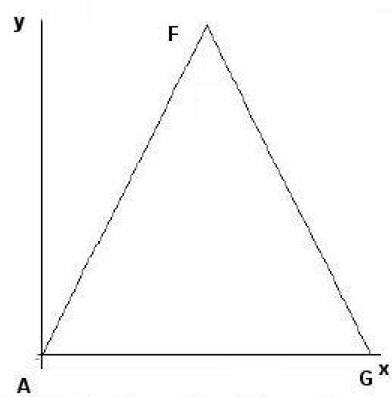

Figure 2: Symmetrical Space Diagram

$$2\sqrt{(x'^2_F + y_F^2)} = 2\sqrt{[\alpha^2 (x_F - vt_F)^2 + y_F^2]} = 2\, y_F ,$$

since $x_F = vt_F$. Note that the factor α has dropped out. The average speed for the return trip is

$$< c > = 2\, y_F / (2\, t_F) = \sqrt{(1 - v^2)} , \qquad (7)$$

contradicting the requirement that this should be unity. The conclusion is that the adjustment factor α in Eq.(4) is not up to the job of keeping the observed return light speed equal to unity. We need to impose an adjustment in the y- and z-directions, perpendicular to the direction of motion of the observer. The adjusted average speed will be $< c > = y'_F / t_F$, where $y'_F = \beta y_F$, with

$$\beta = 1 / \sqrt{(1 - v^2)} . \qquad (8)$$

Since $\alpha = \beta^2$, the proposed transformation is

$$x' = \beta^2(x - vt) \qquad y' = \beta y \qquad z' = \beta z \qquad t' = t . \qquad (9)$$

Note that there is no modification of the time: it is *still* the absolute time of Newton. The transformation (9) amounts to a contraction in the direction of motion, and a lesser contraction in the perpendicular directions.

We shall now show that this transformation works also for an asymmetrical situation of the sort depicted in Fig. 3, in which the light travels forwards and backwards in a skewed manner, relative to the motion of the observer through the ether. With respect to S, in which the ether is at rest, the speed of light is one in all directions, so

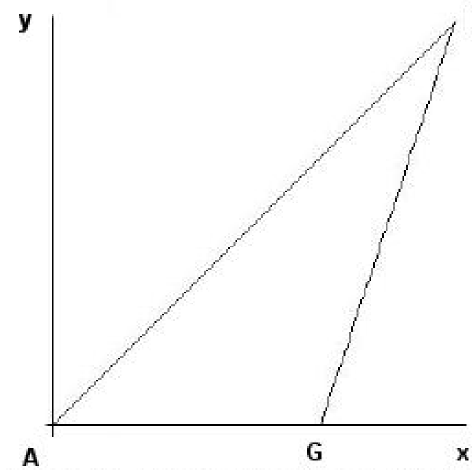

Figure 3: Asymmetrical Space Diagram

$$x_F^2 + y_F^2 = t_F^2$$
$$(x_F - x_G)^2 + y_F^2 = (t_G - t_F)^2 \ .$$

Subtract one equation from the other, substitute $x_G = vt_G$, and then cancel a factor of t_G throughout the equation. One finds

$$t_F - vx_F = (1 - v^2)\, t_G / 2 = t_G / (2\, \beta^2) \ . \qquad (10)$$

The average return speed, as measured by the moving observer, is

$$< c > = \{\sqrt{\ } [x'_F{}^2 + y'_F{}^2] + \sqrt{\ } [(x'_F - x'_G)^2 + y'_F{}^2] \} / t_G = 2 \{\sqrt{\ } (x'_F{}^2 + y'_F{}^2) \} / t_G \ . \qquad (11)$$

Now $y'_F{}^2 = \beta^2\, y_F{}^2 = \beta^2 (t_F{}^2 - x_F{}^2)$, and some algebra produces

$$x'_F{}^2 + y'_F{}^2 = \beta^4\, (x_F - vt_F)^2 + \beta^2\, (t_F{}^2 - x_F{}^2) = \beta^4\, (t_F - vx_F)^2 \ ,$$

and so Eqs. (10)-(11) yield the average velocity

$$< c > = 2\, \beta^2 (t_F - vx_F) / t_G = 1 \ .$$

We conclude that the transformation (9) indeed preserves the invariance of the average return speed of light that is measured by the moving observer, irrespective of the angle between the directions of propagation of the light and the motion of the observer through the ether.

3. Relativity of Time

Eq. (9) is by no means the most general transformation of the spacetime coordinates that leaves the two-way speed of light unchanged. Introduce first a common scaling factor, λ, in the definition of x', y', z' and t', which clearly leaves all speeds unchanged:

$$x' = \lambda\, \beta^2 (x - vt) \qquad y' = \lambda\, \beta\, y \qquad z' = \lambda\, \beta\, z \qquad t' = \lambda\, t \ . \qquad (12)$$

This transformation also preserves the two-way speed of light in the general situation depicted in Fig. 3, for in place of Eq. (11), we now have for the average speed of light, as measured by the observer in S',

$$< c > =2 \{\sqrt{\ } (x'_F{}^2 + y'_F{}^2) \}/ t'_G = 1 \ . \qquad (13)$$

Note that this is the first point in the development that the possibility has been entertained that time might not be absolute, to the extent that there is a scaling between time in S and time in S'. The transformation (12) may be further generalized through the addition of any multiple of x – vt to the definition of t', since $x_G - v\, t_G = 0$, so if we replace Eq.(12) by

$$x' = \lambda\beta^2 (x - vt) \qquad y' = \lambda\beta\, y \qquad z' = \lambda\beta\, z \qquad t' = \lambda t + \varepsilon\, (x - vt) \ , \qquad (14)$$

then neither t'_G nor the average return speed of light will be changed. The relation between time t in S and t' in S' is now more complicated. Whereas simultaneity is absolute for the transformation (12), that is no longer the case for the transformation (14), on condition that ε is nonzero. That is, for two points (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2), for which $t_1 = t_2$ but x_1 is not equal to x_2, $t'_1 = t'_2$ under (12) but not under (14).

We shall now relate the parameters λ and ε by the method of *transport synchronization* of clocks [Mansouri and Sexl 1976-77]. This technique is intuitively the most convincing way of ensuring that clocks at different places tell the same time. In the frame S, at rest with respect to the ether, we suppose that there are many clocks, all of the same constitution, situated at different points of space. Take a clock at any point other than the origin and transport it to the origin, where there is a clock that will be used as standard. Reset the clock that has been moved so that it agrees with the clock at the origin, and then take the first clock back to its original place *very slowly*, that is, by imparting a very small velocity to it in the right direction, and then bringing it to rest. The procedure is repeated for many clocks that are situated at different points in space, so that finally one has a whole lattice of clocks that tell the same time t, at various spatial points (x, y, z).

Suppose that there are also many clocks that all have the same speed v parallel to the x-axis, so they are all at rest in S'. The spacetime point $t = x = y = z = 0$ coincides with $t' = x' = y' = z' = 0$, so the clock at the origin of S' passes the clock at the origin of S at time zero, at which instant the former clock is set to zero. The other clocks at rest in S' are now synchronized with the clock at the origin of S' by transport synchronization, that is to say they are brought to the origin of S', reset to agree with the clock there, and then brought back very slowly to their original spatial point in S'. The question now arises: when a synchronized clock in S' passes by a synchronized clock in S, do they agree? That depends on the choice of the parameter ε in Eq. (14). The specification of the procedure of transport synchronization consists in requiring that ε should be restricted so that indeed $t' = t$ at all points of space. Let us see how that works in detail.

Imagine that one of the S' clocks on the x'-axis has been brought to the origin of S', reset to agree with the clock there, and then moved back slowly to its original position. To do that we give it a speed u, *as measured from S,* which is infinitesimally larger or smaller than v, so that the clock indeed moves at a snail's pace, as seen from S'. The worldline of this clock is $x = ut$, which means that the time at the clock's position, as measured in S', is $t' = \lambda[v]\, t + \varepsilon[v]\, (u - v)\, t$, where the dependence of λ and ε on v has been indicated explicitly. Define a new frame of reference S'', such that our clock remains at rest at the origin, in which the spacetime coordinates (t'', x'', y'', z'') are defined as (t', x', y', z') were defined in Eq.(14), excepting only that v is replaced by u. The time in S'', the rest-frame of our clock, is $t'' = \lambda[u]\, t$, where λ now is the value of that parameter associated with the velocity u rather than v. Note that the ε-term does not contribute to t''. Transport synchronization requires t'' to be equal to t', so

$$\lambda[u]\, t = \{\lambda[v] + \varepsilon[v]\, (u - v)\}\, t, \qquad \text{which means} \qquad \varepsilon[v] = \{\lambda[u] - \lambda[v]\} / (u - v) .$$

Strictly, this equality is supposed to be true only in the limit of infinitesimal $(u - v)$, so we conclude that $\varepsilon[v] = \lambda'[v]$, where the prime indicates differentiation with respect to v. Thus the general transformation (14), with transport synchronization implemented, reads

$$x' = \lambda\beta^2(x - vt) \qquad y' = \lambda\beta\, y \qquad z' = \lambda\beta\, z \qquad t' = \lambda t + \lambda' (x - vt) . \tag{15}$$

The scaling factor λ depends on the velocity v, but not on its sign, as can be seen by imagining a rotation of the whole system by 180 degrees about the z-axis, which effects a change of v into – v, but no change of measurements in the z-direction, so necessarily $\lambda[-v] = \lambda[v]$.

4. Vindication of Einstein

Close to the beginning of Einstein's momentous first paper on the theory of relativity, he writes

> The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties …" [Einstein 1905]

A consequence of the assumption of the equivalence of all inertial systems, excluding any special significance for one frame, is that one could equally well regard the inertial frame S' as being the one in which light has unit speed in all directions, and S as the one in which the return speed of light is constant. The inverse transformation from S' to S must have the same form as (15), except that (t', x', y', z') and (t, x, y, z) are interchanged, and v is replaced by – v . As already shown, λ is not changed by this replacement. Now imagine the successive implementations of the transformation from S to S', and then from S' back to S. This should be equivalent to the identity, i.e. to no transformation at all, but since the double transformation has the effect $y' = \lambda\beta y$, followed by $y = \lambda\beta y' = \lambda^2\beta^2 y$, it must be true that $\lambda^2\beta^2 = 1$. The equality must hold continuously in the limit in which v tends to zero, so $\lambda\beta = 1$, or $\lambda = 1 / \beta = \sqrt{(1 - v^2)}$. Moreover, the derivative of λ with respect to v is $\lambda' = - v/\sqrt{(1 - v^2)} = - v\,\beta$, and so the transformation (15) takes on the form

$$x' = \beta\,(x - vt) \qquad y' = y \qquad z' = z \qquad t' = t / \beta - v\,\beta\,(x - vt)\,. \tag{16}$$

Since $t / \beta + \beta\, v^2\, t = \beta\, t\, (\beta^{-2} + v^2) = \beta\, t$, this relation can be rewritten

$$x' = \beta\,(x - vt) \qquad y' = y \qquad z' = z \qquad t' = \beta\,(t - v\,x)\,, \tag{17}$$

which is the Lorentz transformation, equivalent to the form in which Einstein wrote it down in 1905. One year earlier, Lorentz had derived the same relation [Lorentz 1904], first with an extra scaling factor, which he then proved to be equal to unity.

In his derivation of the Lorentz transformation, Einstein did not follow the above steps. Rather he postulated right at the beginning the equivalence of all inertial systems and the constancy of the one-way speed of light, and his calculation is relatively short. He did not employ transport synchronization to determine the time at different spatial points of an inertial system: instead he used the postulate of the invariance of the one-way speed of light to define time at a point of space in terms of the time-of-transmission of light from the origin to the point and back again to the origin. Einstein finally showed that Maxwell's electromagnetic equations are not changed in form if one performs the Lorentz transformation, a more elaborate demonstration which, as we have mentioned, Lorentz had already completed.

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